INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & MANAGEMENT

STABILITY ANALYSIS OF A ROBUST NONLINEAR CONTROLLER FOR FLEXIBLE JOINT ROBOT

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ABSTRACT

In this research one control system has been designed for robot motion in three dimensional spaces with providing an analytic method for preventing strike to obstacle. One kind of mechanism which is aided by combinational method is used to determine a mechanical arm route and to reach the optimum responses quickly. So first the rigid robot dynamics and its PID control is considered, then FJR becomes a model with structural and nonstructural indefiniteness and changes to the standard form of the resistant control theory. Then the proposed control algorithm is presented given PID control of the rigid robot. Theresults survey and their comparison with normal or ideal states show that we could approach an ideal condition by this system help. Finally, mathematical details of algorithm resistant consistency proof are expressed with two propositions and an adequate condition is obtained for resistant consistency of system.

Keyword: Robots with flexible joints, Harmonic drive, Resistant PID, UUB consistency.

INTRODUCTION

In nonlinear system, one qualitative measure for desirable state in requested work area is defined in which we use computer simulation as the most appropriate tool to be certain that given qualitative measure is met.

The robotic systems are one of the nonlinear systems. With recent robotic industries development, the standard usages of the industrial robots have been increased in new different fields. The robots are one of the best options in the industrial automation. In the environments in which there is a little security, the robots can be a proper substitute for human elements. High repeatability, planning, and precision of performance are the robots essential features. One of the robots kinds in terms of exterior form is “robot skillful hand” which has many usages in the different industries. The industrial robots are mainly referred to as “robot skillful hand” which do tasks such as picking up and attaching pieces, welding, dyeing, mounting, and installing different parts of one machine, etc. in fact one of the most essential issues in robotics is to design a mechanical system in order to move cargo precisely. While there are wide usages in this context, there is a common necessary need in all of them which are as follows.

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The importance of considering flexibility in modeling and the industrial robots control has been shown empirically in the different articles. On the other hand, the harmonic drives and torque transformers have been widely used in the industrial and spatial robots. The flexibility resulted from these elements is one of the reasons why controlling robots with flexible joints are widely regarded during recent decades.

Recent years many research have been done on controlling robots with flexible joints. Moreover, the basic problem of these robots control and implementation is an innate flexibility relating to robots joints. The robot control with the flexible joints is an important category to which the researchers have paid attention recently. Strops, long axises, harmonic drives, etc., in the industrial robots structure cause the problem we couldn’t consider the rigid model for them. As a result, we should use the flexible model – based methods for controlling the industrial robots control. Many researchers have regarded to the joints flexibility as the one of the most important in definiteness of FJR.

Because of joints flexibility, propellant situation (e.g. motor axis angle) is not directly related to drive axis situation and this is not proper in the very precious usages at all. Meanwhile undesirable oscillation resulted from joints flexibility imposes the band width limitation on the total control algorithm based on the designed rigid robots and may cause the consistency problem for control rules which ignore the joints flexibility effect.

Traditionally, many researchers would pay attention to controlling systems with cinematic closed loop because of dynamic equations complexity and cinematic obligations of joint variables. Most of the control methods which have proposed for robots such as nonlinear methods that are used for robots control.
This study tries to investigate the resistant consistency of controllers in robots with the flexible joints. One new method for resistant control of these robots will be provided. So first the rigid robot dynamics and its PID control is considered, then FJR becomes a model with structural and nonstructural indefiniteness and changes to the standard form of the resistant control theory. Then the proposed control algorithm is presented given PID control of the rigid robot.
This controller consists of one resistant PID based on the rigid robot and one revisory term which is added to rectify the joints flexibility effect. The simplicity and linearity are of this controller features. Then mathematical details of the proposed algorithm resistant consistency proof are expressed with two propositions and an adequate condition is obtained for resistant consistency of system. Finally, the proposed algorithm reliability with simulation of biaxial arm with flexible joint is considered.

MODELING

One common robot system shown in Fig. 1 consists of one motor which is led by a control system and one transition system attached to it including gearbox and ball screw which converts revolving motion to linear one and its output force is shifted to cargo by a spring.

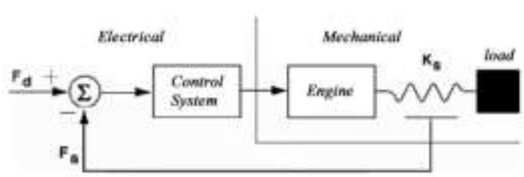

Figure1: Schematic of robot mechanism

By metering the spring length in this mechanism, we can control the output force amount at the robot end. In this model it is supposed that the dynamic response of system is as quick as enough.
Figure2 shows one model of this mechanism whose state will be modeled by Matlab Simulink Software in the following.

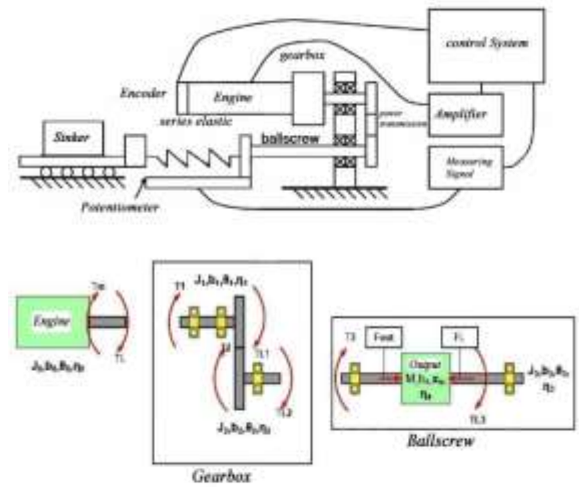

Figure2: Series elastic mechanism

Given figure 2, the dynamic equation of this system can be written as follows:

$$\begin{aligned}
&J_0\ddot{\theta}_0 + b_0\dot{\theta}_0 + T_L = \eta_0 T_m \\
&J_1\ddot{\theta}_1 + b_1\dot{\theta}_1 + T_{L1} = \eta_1 T_1 \\
&J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 + T_{L2} = \eta_2 T_2 \\
&J_3\ddot{\theta}_3 + b_3\dot{\theta}_3 + T_{L3} = \eta_3 T_3 \\
&M\ddot{x}_s + b_4\dot{x}_s + F_L = \eta_4 F \\
&x = \frac{P\theta_3}{2\pi}
\end{aligned} \tag{1}$$

Where J_i ، b_i ، θ_i ، i، η_{Ti} and, T_{Li} are Moment of inertia, damping coefficient, revolving situation, efficiency, input torque, and output torque of i*th* member, alternatively and p is ball screw step according to figure 2.
Also, we can set the following relation given mechanical performance of mechanism:

$$\begin{aligned}
&T_{L3} = K_L F_L \\
&T_L = T_1
\end{aligned} \tag{2}$$

$$T_{L1}\theta_1\eta_G = T_2\theta_2 \Rightarrow T_{L1} = \frac{\theta_2}{\theta_1}\frac{T_2}{\eta_G} \Rightarrow T_{L1} = K_G\frac{T_2}{\eta_G}$$

$$T_3 = T_{L3}$$

Where K_L is cargo transition coefficient of ball screw, Gη is power transition efficiency in gearbox, and K_G is power transition ratio in gearbox.
Moreover, there are following relations based on geometrical obligations:

$$\begin{aligned}
&\theta_0 = \theta_1 \\
&\theta_2 = K_G\theta_1 \\
&\theta_3 = \theta_2
\end{aligned} \tag{3}$$

According to above equations, we will have:

$$T_L=\left(\frac{J_1}{\eta_1}+\frac{K_G^2 J_2}{\eta_1\eta_2\eta_G}+\frac{K_G^2 J_3}{\eta_1\eta_2\eta_3\eta_G}\right)\ddot{\theta}_0 \qquad (4)$$
$$+\left(\frac{b_1}{\eta_1}+\frac{K_G^2 b_2}{\eta_1\eta_2\eta_G}+\frac{K_G^2 b_3}{\eta_1\eta_2\eta_3\eta_G}\right)\dot{\theta}_0+\frac{K_G}{\eta_1\eta_2\eta_3\eta_G}T_{L_3}$$

In the proposed scheme of simulation issue, used motor is a DC one whose schematic performance is shown in Figure3.

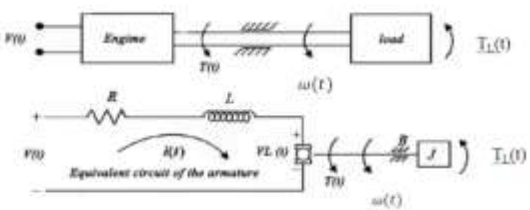

Figure3: Equivalent circuit of dc motor

Given motor equivalent circuit, dominant equations on motor are calculated as follows:

$$v(t)=Ri(t)+L\frac{d}{dt}i(t)+v_b(t)$$
$$v_b(t)=K_b\dot{\theta}_0 \qquad (5)$$
$$T_m(t)=K_T i(t)$$
$$J_0\ddot{\theta}_0+b\dot{\theta}_0=T_m(t)+T_L(t)$$

Using figure 3 and also equations of (5), the motor block diagram is in the form of Figure4.

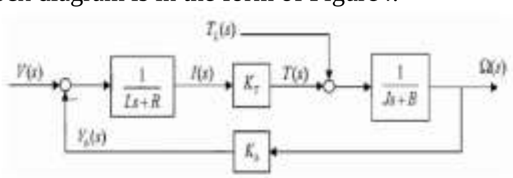

Figure4: Block diagram dc motor function

For applying Simolink Software, it is necessary to translate the obtained equations for mechanism with common concepts of the mentioned software. On the other word, we should model it to software language. Figure4 shows the basic part of this model in which motor output entered to power transition system, is applied to series robot by following obtained equations. This system is controlled by one PID controller help in the form of a closed loop. Robinson has done some researches on how to design the given controller and considered it in details [16]. But in this study, we obtained empirically the needed coefficients for controller.

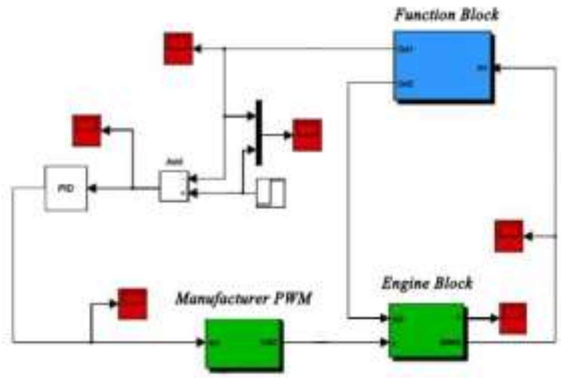

Figure5: General model system controller

The block diagram Figure4 shows motor block diagram which forms motor block in Figure5 input voltage to motor is controlled by controller using PWM wave producer which is brought in Figure5 has been mapped in Figure6 with more details.

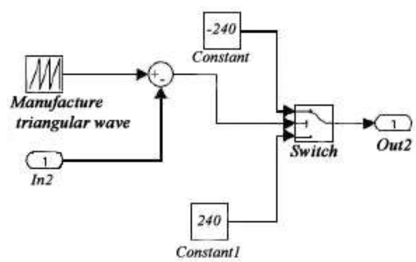

Figure6: PWM wave generator

The sub-block of operator which is shown in Figure5 includes power transition system and operator output (in which revolving motion of operator changes to linear motion). Figure7 shows this sub-block details.

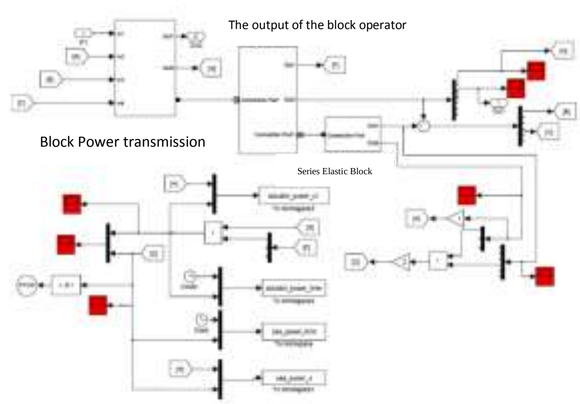

Figure7: The following details the Robot Block

As it has been shown in Figure7, this sub-block has consisted of 3 other sub-blocks for power transition, operator output, and robot. The sub-block of power transition in which motor revolving motion changes to linear motion by gearbox and ball screw, has been shown in Figure8.

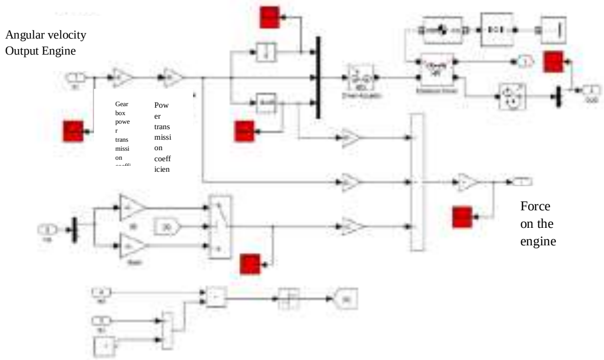

Figure8: Block diagram of the power transmission system

According to Figure2 Output power by a mass transit system (The mass of the Ball Screw) In fact, the total output power operator Can handle is attached to the robot. The following function block output power in Figure7 It defines the crime More details are given in Figure9.

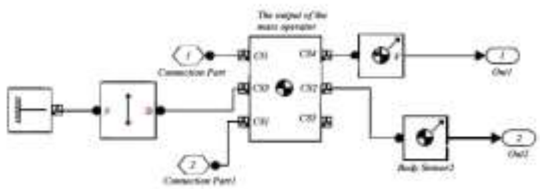

Figure9: The following function block output power supply

The last section of this system is robot. Given loading at the end of robot, we can model this section to two forms. First type which is at the end of open systems and second type which is at the end of closed (limited) system are shown in figure 10, alternatively.

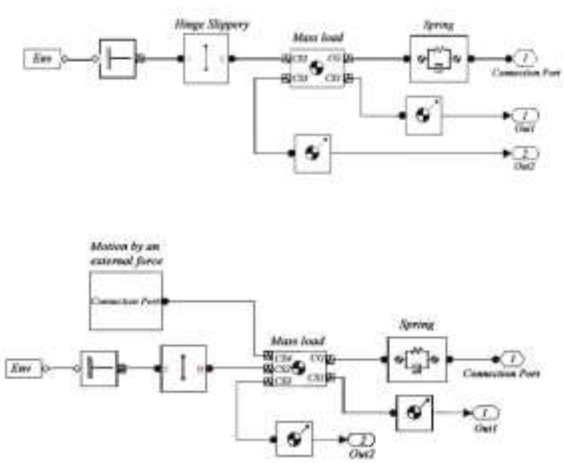

Figure 10: Series elastic model. Above the bottom of the open loop closed loop

THE EFFECT ON THE PERFORMANCE OF ROBOT BEHAVIOR TO MAKE OPERATOR

Here, by considering changes of robot spring rigidity and robot output power estimation in different condition and this power comparison with power production of a non-series elastic operator, it is shown that adding one elastic factor such as spring to the operator can lead to gradual increase of power production at its output. Two systems are considered in this research.

4-1- Robot in the open-loop condition

Given condition for this investigation is like a condition in which there are normal manner when jumping or entering to swing phase in stepping motion. In this condition power increase in joints and when sped reaches to the extent that the necessary power for given motion is supplied, member attached to the joint is thrown like a projectile.

For modeling this condition, the effect of robot adding on robot and operator power production is considered by bringing operator replacement to the certain extent in that is the same maximum operator course. Figure11 shows this situation schematically.

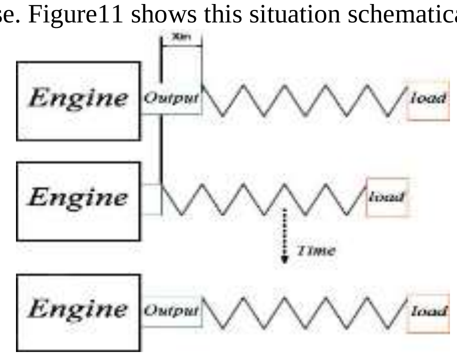

Figure11: Robots symbolically in terms of the open loop

Figure12 shows output power of operator and output power of robot. As you see, by increasing elastic series rigidity coefficient, maximum output power of operator and robot increase and also output power of robot increase relative to output power of operator in the certain rigidity domain and this is obvious more based on Table1 as the diagrams indicate, by more rigidity increase in the elastic factor, system output power approaches to power state in which there is no elastic factor between cargo and operator. We can justify this in the way that rigidity increase of relation between cargo and operator leads to rigid relation, as a result this situation is like a state that the cargo is directly attached to operator.

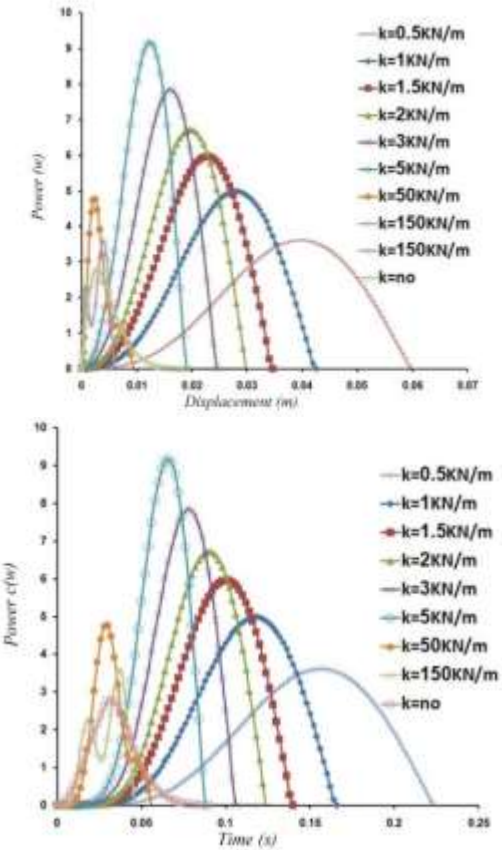

Figure12:The output power of the operator and the robot in terms of open-loop

Table 1. The output power and operation of robots with different stiffness in open loop

Stiffness (KN/m)	*Output Power operator*	*Output power series elastic*	*Increase the output power to the robot operator output*	*Percent increase in output power to the robot operator*
0.5	2.8	3.6	0.8	28.6
1	3.9	5	1.1	28.2
1.5	4.7	6	1.3	27.7
2	5.3	6.8	1.5	28.3
3	6.3	7.9	1.6	25.4
5	7.5	9.2	1.7	22.7
50	4.2	4.8	0.6	14.3
150	3.6	3.6	0	0
Rigid	2.8	2.8	0	0

The obtained results of above experiment can be considered in other viewpoint. Figure 12 shows this investigation. Given Figure 13. A and Figure 13. D, the robot rigidity increase causes time decline for reaching maximum power in operator output and robot end and also time for reaching maximum power in operator output is less than given time at robot output for certain rigidity domain. Figure 13. B shows that rigidity increase leads to decrease necessary displacement for approaching maximum power at robot output and operator attached to it. This is more obvious in Figure 13. C.

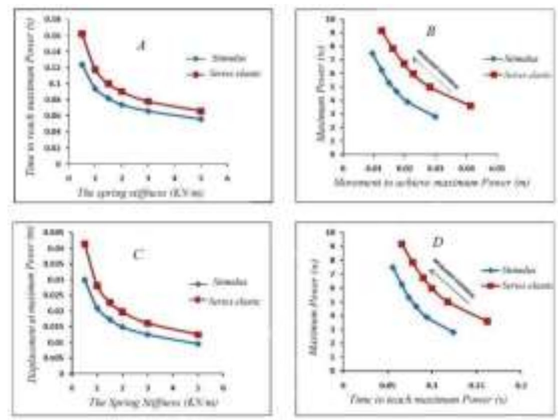

Figure 13: The effect of elastic stiffness on the maximum amount of time to reach maximum power handling required to achieve maximum power

4-2- Robot in the closed-loop condition

This model in which attached cargo to robot or operator is under a compulsory displacement or compulsory force in biomechanical systems, would be similar to the state in which normal situation of stepping is in stuns situation. In this condition, wrist joint preserves its situation at a certain angle by torque production. In the simulated model, the elastic series operator bounds two arms of one hinge joint to each other which can be an example of wrist joint in TBYAL Trans protes. This bound is so that the elastic series operator is obliged to create an angle displacement equal to measured displacement in natural wrist joint in one motion cycle. Also, operator output displacement continues until torque production at robot end becomes equal to the natural foot wrist torque according to 6-3 relations. This displacement has been shown in diagram.

We can calculate the necessary displacement for approaching torque at robot end to needed torque at the foot wrist by comparing power production in operator with needed power at natural foot wrist for different rigidities of robot in these operator types.

$$\tau_{Ankle} = (\theta_{motor} - \theta_{Ankle})\, K_s \qquad (6)$$

Figure 15 shows this survey results. As you see in diagram, the rigidity coefficient increase in the elastic factor leads to decrease the necessary maximum output power in operator for bringing torque production at output of robot relative to the natural torque.

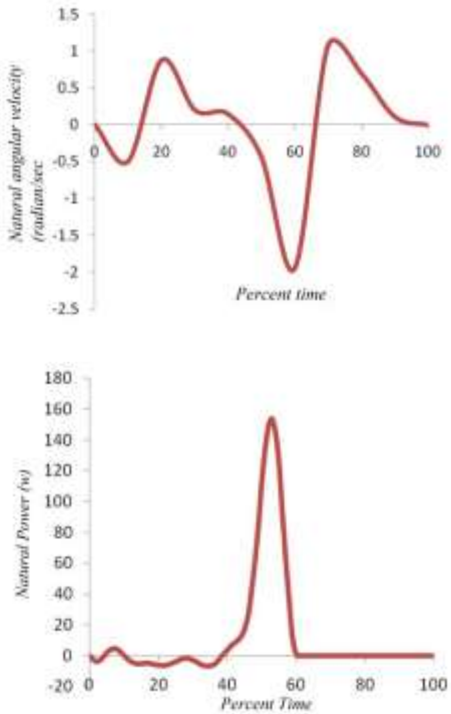

Figure 14: details the natural biomechanics

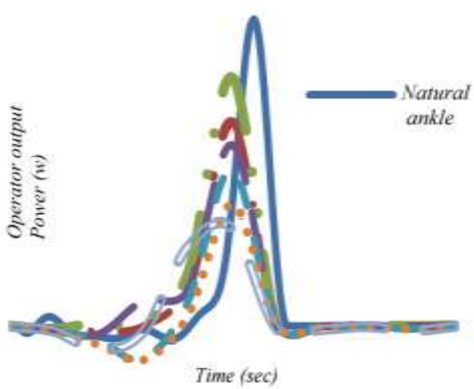

Figure 15:The power output of the operator and shift operator, for the rigidity of the robot in closed loop

Figure 16 shows the angle displacement at operator output in terms of different rigidities of the elastic factor. When the rigidity increase in this factor, we can see that operator angle change get close to measured extent of wrist angle.

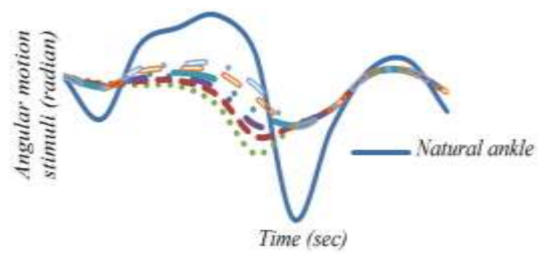

Figure 16:Output shift operator in terms of the ratio of the closed-loop

The obtained results of above experiment can be expressed in the other form. As Figure 17 diagrams indicate, work extent which is done by operator decreases when the rigidity of robot increase and this can be a profit for adding robot to operator.

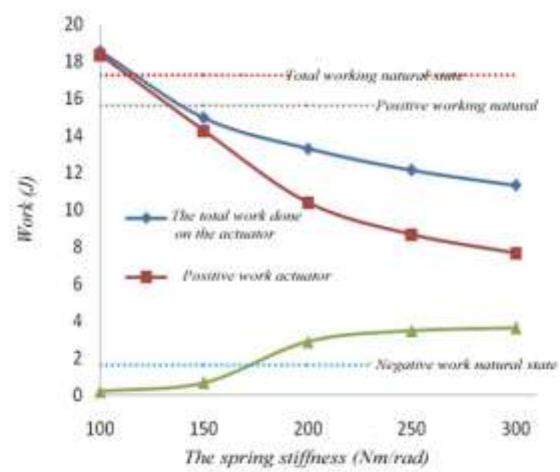

Figure 17: Work performed by the operator in terms of closed-loop

PROOF OF STABILITY

Given the total model, the robot dynamics of rigid axises is as follows [10]:

$$M_t(q)\ddot{q} + N_t(q,\dot{q}) = u_0 \tag{7}$$

Where

$$\begin{cases} M_t(q) = M(q) + J \\ N_t(q,\dot{q}) = V_m(q,\dot{q})\dot{q} + G(q) + F_d(\dot{q}) + T_d \end{cases} \tag{8}$$

M(q) In fact in above eqautions, matrix n*n is $V_m(q,\dot{q})$,matrix n*n includes coriolis and centrifuge, G(q) is n*1 vector of gravity, f_d diametrical matrix of n*n viscose، friction$V_m(q,\dot{q})$ constants, $F_s(\dot{q})$ is n*1 vector of khak friction term (coilomb), T_d is n*1 vector of turbulence or unmodeled but limited dynamic effects, and j is diametrical matrix of n*n propellant inertia. As you see in [10] and [11], in spite of the indefiniteness in all parameters, the equation are as follows:

$$\begin{gathered} m_t I \le M_t(q) \le \overline{m}_t I \\ \|N_t\| \le B_o + B_1\|L\| + B_2\|L\|^2 \\ \|V_m\| \le B_3 + B_4\|L\| \end{gathered} \tag{9}$$

Where $\underline{m}_t, \overline{m}_t, \beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ are the actual positive constants and $\|.\|, L = e^T e^T$ indicates the Euclid term. If we consider u_0 amount as follows:

$$u_0 = KVe + \dot{K}Pe + K_I\int_0^t e(s)ds = Kx \tag{10}$$

Where each of amounts is:

$$\begin{gathered} e = qd - q \\ K = [K_I \quad K_P \quad K_V] \\ x = \left[\int_0^t e^T(s)ds \quad e^T \quad e^{-T}\right]^T \end{gathered} \tag{11}$$

Now place these amounts in the following equation:

$$M_t(q)\ddot{q} + N_t(q,\dot{q}) = u_0 \tag{12}$$

The equation will be as follows:

$$\dot{x} = Ax + B\Delta A \tag{13}$$

That in fact A is matrix as follows:

$$A = \begin{bmatrix} \theta & I_n & \theta \\ \theta & \theta & I_n \\ -M_t^{-1}K_I & -M_t^{-1}K_P & -M_t^{-1}KV \end{bmatrix} \tag{14}$$

And B is matrix like this:

$$B = \begin{bmatrix} \theta \\ \theta \\ M_t^{-1} \end{bmatrix} \text{ و } \Delta A = N_t + M_t\ddot{q}_d \tag{15}$$

We consider the robot manner under the rigidity condition and complete condition of system. As a result we can consider the control rule as a following form.

$$u = u_2 + K_d(\dot{q}_1 - \dot{q}_2) \tag{16}$$

In this relation, U_r is PID control which is given by (4) and K_d is constant diametrical matrix that its elements are from $o(1/\varepsilon)$ order.

By placing the control rule of (16) in the following relation,

$$\begin{cases} M_t(q1)q1 + N(q1,q1) = K(q2 - q1) \\ Jq2 = K(q1 - q2) + u \end{cases} \tag{17}$$

And defining variable Z in the form of,

$$z = K(q_3 - q_1) \tag{18}$$

We will have:

$$J\ddot{z} + K_d\dot{z} + Kz = K(u_2 - J\ddot{q}_1) \tag{19}$$

Given our supposition about K and choosing K_d from $o(1/\varepsilon)$ order we can write:

$$K = \frac{K_1}{\varepsilon^2}, K_d = \frac{K_2}{\varepsilon} \tag{20}$$

Where k1 and k2 are from o(1) order. We can rewrite relation (19) like this:

$$\varepsilon^2 J\ddot{z} + \varepsilon K_d z + K_1 z = K_1(u_2 - J\ddot{q}_1) \tag{21}$$

Now the equation of (17) can be written as follows:

$$\begin{cases} M(q)\ddot{q} + N_t(q,\dot{q}) = z \\ \varepsilon^2 J\ddot{z} + \varepsilon K_2 z + K_1 z = K_1(u_2 - J\ddot{q}_1) \end{cases} \tag{22}$$

System (22) is a system with exceptional deviances whose slow variables are $q_1, \dot{q}_1$ parameters of axis and its fast parameters are $\dot{z}, z$ variables.

Using the results of the exceptional deviances theory, we can estimate the flexible system (22) into two quasi-steady state system and boundary layer system. With $\varepsilon = 0$ For equation (22) we will have:

$$\bar{z} = \bar{u}_2 - J\bar{\ddot{q}}_1 \tag{23}$$

That indicates the variables definition in $\varepsilon = 0$. By placing (23) in (22) we will have:

(24)

$$(M(\overline{q_1}) + J)\ddot{\overline{q}}_1 + N(\overline{q_1}, \dot{\overline{q_1}}) = \overline{u}_2$$

This equation which is similar to rigid model of robot with $\overline{q}_1$ variable, the system is called quasi-steady state system.

Using Tikhonov proposition [3], the elastic force of joints z(t) and angle of axis q(t) for t>0, provide these conditions:

$$\begin{cases} z(t) = \bar{z}(t) + \eta(\tau) + o(\varepsilon) \\ q_1 = \overline{q_1}(t) + o(\varepsilon) \end{cases} \quad (25)$$

Where $\tau = t/\varepsilon$ is fast time scale and η holds true in boundary layer equation:

$$J\frac{d^2\eta}{d\tau^2} + K_2\frac{d\eta}{d\tau} + K_1\eta = 0 \quad (26)$$

Given these results, we can estimate system with flexible joint (23) up to $o(\varepsilon)$ order like this:

$$\begin{cases} (M(q_1) + J)\ddot{q}_1 + N(q_1, \dot{q}_1) = \overline{u}_2 + \eta(\frac{t}{\varepsilon}) \\ J\frac{d^2\eta}{d\tau^2} + K_2\frac{d\eta}{d\tau} + K_1\eta = 0 \end{cases} \quad (27)$$

Because we can choose k_2 Gain properly so that the constant boundary layer system becomes asymptotic, then for very small amount of ε, the flexible system response with rigid U_r control (PID) addition to the revisory term $K_d(\dot{q}_1 - \dot{q}_2)$ can approach after initial decline of quick variables responding which is indicated with $\eta(\tau)$, that is the rigid system controlled by U_r only.

We considered PID control of consistency and rigidity model in the last section. Also, it was shown that the boundary layer system get consistent asymptotically under the revisory term effects. As we know, generally based on consistency of two boundary layer subsystem and quasi-steady subsystem, we can't judge about complete system consistency [3]. According to obtained results of previous sections, we will discuss about complete system consistency and verify its consistency which is UUB type. So we rewrite dynamic equations dominant on FJR:

$$\begin{cases} (M(q_1) + J)\ddot{q}_1 + N(q_1, \dot{q}_1) = u_r + \eta(t/\varepsilon) \\ J\frac{d^2\eta}{d\tau^2} + K_2\frac{d\eta}{d\tau} + K_1\eta = 0 \end{cases} \quad (28)$$

By placing U_R OF (10) and given $e = q_d - q_1$, we will have:

(29)

$$\begin{matrix} e \\ \dot{e} \\ \ddot{e} \end{matrix} = \begin{matrix} \theta & I & \theta \\ \theta & \theta & I \\ M_t^1 K_1 & M_t^1 K_1 & M_t^1 K_V \end{matrix}$$

$$\begin{bmatrix} \int_0^t e(s)d(s) \\ e \\ e \end{bmatrix} + \begin{bmatrix} \theta \\ \theta \\ M_t^{-1} \end{bmatrix}(N_t + M_{tq\ddot{d}}) + \quad (30)$$
$$\begin{bmatrix} \theta \\ \theta \\ -M_t^{-1} \end{bmatrix}\eta$$

Likewise,

$$\begin{matrix} \dot{\eta} \\ \ddot{\eta} \end{matrix} = \begin{matrix} \theta & I \\ J_K^1 & J_{K_d}^1 \end{matrix} \begin{matrix} \eta \\ \eta \end{matrix} \quad (31)$$

Considering the

$$x = \int_0^t e(s)tds\ e^T \dot{e}^{T^T}$$

and

$$y = \eta^T \eta^{T^T}$$

We will:

$$\dot{x} = Ax + B\Delta A + C[I \quad \theta]y \quad (32)$$

$$\dot{y} = \overline{A}y \quad (33)$$

That

$$A = \begin{bmatrix} \theta & I & \theta \\ \theta & \theta & I \\ -M_t^{-1}K_I & -M_t^{-1}K_P & -M_t^{-1}KV \end{bmatrix} \quad (34)$$

$$C = \begin{bmatrix} \theta \\ \theta \\ -M_t^{-1} \end{bmatrix}, \tilde{A} = \begin{bmatrix} \theta & I \\ -J^{-1}K & -J^{-1}K_d \end{bmatrix} \quad (35)$$

Proposition 1: there is the certain diametrical and positive matrix of K_d so that closed loop system described by (33) will be comprehensive and consistent asymptotically.

Proof: we consider chosen Liapanof function as follows:

$$V_F = -y^T W_y, W = \begin{bmatrix} K & 0 \\ 0 & k_d - J \end{bmatrix} \quad (36)$$

For the certain positive constant of S, it is enough to $K_d > J$, now by derivating from V_F along with (33) response, we will have:

$$\dot{V}_F = \dot{y}^T S_y + y^T K\eta - \dot{\eta}^T(K_d - J)\eta \dot{<} 0 \quad (37)$$

Given matrices K, K_d, and J are diametrical and certain positive $\dot{V}_F$ is negative and we can write:

$$V_F = -y^T W_y , W = \begin{bmatrix} K & 0 \\ 0 & k_d - J \end{bmatrix} \quad (38)$$

Proposition2: the closed loop system of (32) and (33) are consistent system of UUB type, if K_d و and ξ_1 for proof, we consider this composite Liapanof function:

$$V(x,y) = x^T px + y^T sy \quad (39)$$

$x^T px$ is chosen Liapanof function for the rigid system and $y^T Sy$ is chosen Liapanof function in proposition 1. According to inequality of Rayleigh-Ritz, we can write:

$$\{\underline{\lambda}(p)\|x\|^2 \leq x^T Px \leq \bar{\lambda}(P)\|x\|^2$$
$$\{\underline{\lambda}(s)\|y\|^2 \leq y^T Sy \leq \bar{\lambda}(s)\|y\|^2 \quad (40)$$

Where $\bar{\lambda}$ و $\underline{\lambda}$ indicate the largest and smallest amounts, alternatively and by adding these inequities, we have:

$$\underline{\lambda}(s)\|y\|^2 + \underline{\lambda}(p)\|x\|^2 \leq V(x,y) \leq \bar{\lambda}(S)\|y\|^2 + \bar{\lambda}(p)\|x\|^2 \quad (41)$$

Complimentary

$$z_t = [\|x\| \quad \|y\|]^T \quad (42)$$

We will:

$$[\|x\| \quad \|y\|] \begin{bmatrix} \underline{\lambda}(p) & 0 \\ 0 & \underline{\lambda}(p) \end{bmatrix} \begin{bmatrix} \|x\| \\ \|y\| \end{bmatrix} \leq V(x,y)$$
$$\leq [\|x\| \quad \|y\|] \begin{bmatrix} \bar{\lambda}(p) & 0 \\ 0 & \bar{\lambda}(s) \end{bmatrix} \begin{bmatrix} \|x\| \\ \|y\| \end{bmatrix} \quad (43)$$

Again by applying Rayleigh-Ritz inequity, we can write:

$$\underline{\lambda}\|Z_t\| \leq V(Z_t)\bar{\lambda}\|Z_t\| \quad (44)$$

That we can conclude as follows by placing in the above equations:

$$\underline{\lambda} = Min\{\underline{\lambda}(P), \underline{\lambda}(S)\} \quad (45)$$
$$\bar{\lambda} = Max\{\bar{\lambda}(P), \bar{\lambda}(S)\}$$

Now by derivating (32) and (33) routes from (39), we have:

$$\dot{V} = 2x^T P\dot{x} + x^T \dot{P} x + 2y^T S\dot{y} \quad (46)$$
$$= [2x^T P(Ax + B\Delta A) + x^T \dot{P} x] + 2x^T PC[I \quad 0]y + 2y^T S\dot{y}$$

Given $\dot{V}(x) \leq \|x\|(\xi_0 - \xi_1\|x\| + \xi_2\|x\|^2$ [13], we can write:

$$2x^T P(Ax + B\Delta A) + x^T \dot{P} x \leq \|x\|(\xi_0 - \xi_1\|x\| + \xi_2\|x\|^2 \quad (47)$$

Also by defining $\gamma_1 = \lambda_{max}(M_t)$, we have:

$$2x^T PC[I \quad 0]y \leq 2\gamma_1 \overline{\lambda(P)\|x\|\|y\|} \quad (48)$$

As we see in proposition 1:

$$2y^T S\dot{y} \leq -\lambda_{min}(W)\|y\|^2 \quad (49)$$

So that we can write:

$$\dot{V} \leq [\|x\| \quad \|y\|] \begin{bmatrix} \xi_1 & -\gamma_1\bar{\lambda}(p) \\ -\gamma_1\bar{\lambda}(p) & \lambda_{min}(W) \end{bmatrix} \begin{bmatrix} \|x\| \\ \|y\| \end{bmatrix} + \xi_0\|x\| + \xi_2\|x\|^3 \quad (50)$$

According to (42) we have:

$$\dot{V} \leq -Z_t^T R Z_t + \xi_0\|Z_t\| + \xi_2\|Z_t\|^3$$

That

$$R = \begin{bmatrix} \xi_1 & -\gamma_1\bar{\lambda}(p) \\ -\gamma_1\bar{\lambda}(p) & \lambda_{min}(W) \end{bmatrix} \quad (51)$$

We should have following relation, in order to R be a certain positive:

$$\lambda_{min}(W) > \frac{\gamma_1^2 \bar{\lambda}^2(P)}{\xi_1} \quad (52)$$

By meeting condition (52) by proper choosing of K_d for fast subsystem, we have:

$$V^{\cdot} = \|Z_t\|(\xi_0 - \lambda_{min}(R) + \xi_2\|Z_t\|^2)$$

Now, given (44) and (53) and also 3-5 from [13], if this condition is met, the system will be in the form of UUB relative to consistent $\Upsilon(0, \acute{d})$, that

$$\acute{d} = \frac{2\xi_0}{\lambda_{min}(R) + \sqrt{\lambda_{min}^2(R) - 4\xi_0\xi_2}} \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}} \quad (53)$$

Will be stable uub:

$$\lambda_{min}(R) > 2\sqrt{\xi_0\xi_2} \quad (54)$$

$$\lambda_{min}^2(R) + \lambda_{min}(R)\sqrt{\lambda_{min}^2(R) - 4\xi_0\xi_2} > 2\xi_0\xi_2\left(1 + \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}}\right)$$

$$\lambda_{min}(R) + \sqrt{\lambda_{min}^2(R) - 4\xi_0\xi_2} > 2\xi_2\|Z_{t0}\| \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}}$$

This condition will simply be met by enlarging $\lambda_{min}(R)$, also we can choose $\lambda_{min}(R)$ large enough, by increasing ξ_1 (which is function of K_p و K_1 و K_V) and enlarging $\lambda_{min}(W)$ (which is influenced by K_d) – in order to meet condition.

SUMMARY OF RESULTS FROM ANALYSIS OF SERIES ELASTIC ACTUATORS

In this discussion, we tried to consider the effect of adding the robot to operator at output power produced by operator and robot by modeling one robot in Simolink environment. Here we considered the operator ability as a replacement for muscles and tendons in the natural members, by modeling robot with two different conditions of closed and open loop which can be equal to 2 different states in the biological system. Using these results we can conclude that because of robot similarity to muscle and tendon in the natural members, it is a proper operator for using in protes and natural members designing. Also, adding the elastic factor to the operator can increase the maximum power at the operator output. It is itself an important benefit for this kind operators and make possible to conquest on resistive forces which makes impossible the operator free end in some sections.

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